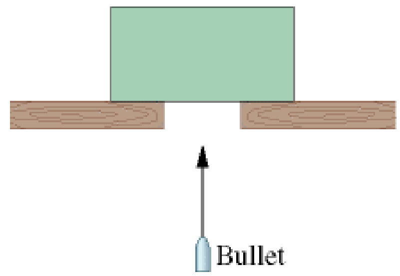
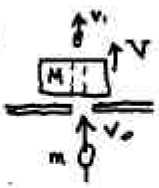
PHYSICS 1401 (1) homework solutions

10-30 A 10 g bullet moving directly upward at 1000 m/s strikes and passes through the center of mass of a 5.0 kg block initially at rest (Fig. 10-33). The bullet emerges from the block moving directly upward at 400 m/s. To what maximum height does the block then rise above its initial position?



10-30



givens: $m = 10\,g = .010\,kg$

$v_0 = 1000\ m/s$

$M = 5.0\ kg$

$v_1 = 400\ m/s$

First, find the velocity of block after the collision using momentum conservation. (Though gravity acts during collision, its impulse is negligible.)

$$m v_0 = m v_1 + M V$$

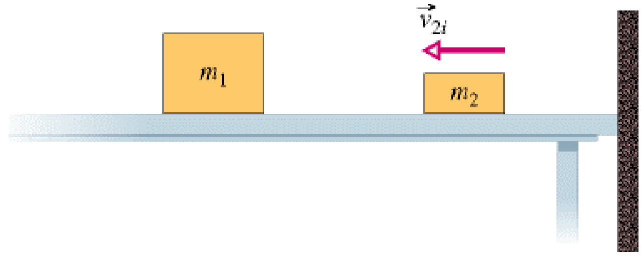
$$V = \frac{m}{M}(v_0 - v_1) = \frac{.010}{5.0}(1000 - 400) = 1.2\ m/s$$

The height, h, attained by the block may be obtained from energy conservation (applied to block only) subsequent to the collision.

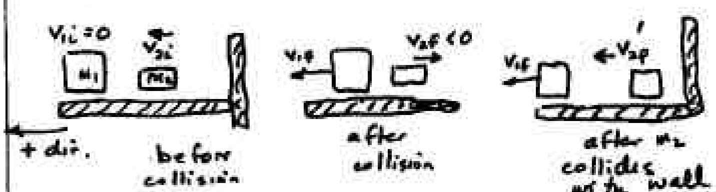
$$\frac{1}{2} M V^2 = M g h \rightarrow h = \frac{V^2}{2g} = \frac{(1.2)^2}{2(9.8)}$$

$$h = .073\ \text{meters}$$

10-44 In Fig. 10-40 , block 1 of mass m_1 is at rest on a long frictionless table that is up against a wall. Block 2 of mass m_2 is placed between block 1 and the wall and sent sliding to the left, toward block 1, with constant speed v^2i. Assuming that all collisions are elastic, find the value of m_2 (in terms of m_1) for which both blocks move with the same velocity after block 2 has collided once with block 1 and once with the wall. Assume the wall to have infinite mass.



10-44



Note: Since m_2 collision is with wall and the wall is ∞ massive
$v'_{2f} = -v_{2f} > 0$

And we are given that $v'_{2f} = v_{1f}$ So $v_{2f} = -v_{1f}$

Writing momentum conservation for collision between blocks

(a) $m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1f} - m_2 v_{1f} = v_{1f}(m_1 - m_2)$

Energy conservation (elastic)

(b) $\frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + m_2 v_{2f}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{1f}^2 = \frac{1}{2} v_{1f}^2 (m_1 + m_2)$

Dividing left-hand side of (a) into lhs of (b) + also rhs's

$$\frac{\frac{1}{2} m_2 v_{2i}^2}{m_2 v_{2i}} = \frac{\frac{1}{2} v_{1f}^2 (m_1 + m_2)}{v_{1f}(m_1 - m_2)} \Rightarrow v_{1f}(m_1 + m_2) = v_{2i}(m_1 - m_2)$$

Replacing v_{2i} with equation (a)

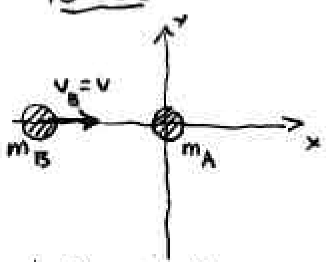
$$v_{1f}(m_1 + m_2) = v_{1f}\frac{(m_1 - m_2)^2}{m_2} \Rightarrow m_1 m_2 + m_2^2 = m_1^2 - 2m_1 m_2 + m_2^2$$

$$3 m_1 m_2 = m_1^2 \Rightarrow \boxed{m_2 = \frac{1}{3} m_1}$$

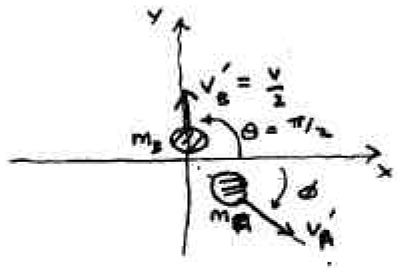
10-50 Two balls A and B, having different but unknown masses, collide. Initially, A is at rest and B has speed v. After the collision, B has speed v/2 and moves perpendicularly to its original motion.

(a) Find the direction in which ball A moves after the collision.
(b) Show that you cannot determine the speed of A from the information given.

10-50



before collision



after collision

Momentum is conserved in both x- and y-directions.
(Note: Since are not told collision is elastic, we cannot assume that mechanical energy conservation.)

x-direction: $m_B v_B = 0 + m_A v_A' \cos\phi$

y-direction $0 = m_B v_B' - m_A v_A' \sin\phi$

Re-writing two equations

$$m_A v_A' \sin\phi = m_B v_B'$$
$$m_A v_A' \cos\phi = m_B v_B$$

(a) Ratio $\tan\phi = \frac{v_B'}{v_B} = \frac{v/2}{v} = \frac{1}{2} \Rightarrow \phi = 26.6°$ (below x-axis)

(b)
We can square and add the two equations

$$m_A^2 (v_A')^2 \left[\sin^2\phi + \cos^2\phi\right] = m_B^2 \left[(v_B')^2 + v_B^2\right]$$

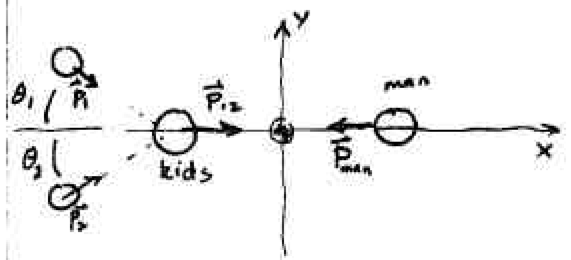
$$v_A' = \frac{m_B}{m_A}\sqrt{\left(\frac{v}{2}\right)^2 + v^2} = v\,\frac{m_B}{m_A}\sqrt{\frac{5}{4}}$$

But we are not given v, or m_B/m_A
So we cannot get v_A'
(with the information provided)

10-54 Two 30 kg children, each with a speed of 4.0 m/s, are sliding on a frictionless frozen pond when they collide and stick together because they have Velcro straps on their jackets. The two children then collide and stick to a 75 kg man who was sliding at 2.0 m/s. After this collision, the three-person composite is stationary. What is the angle between the initial velocity vectors of the two children?

10-54

First choose a coordinate system. Note that we are told that (kids) + (man) stick together at rest. Hence, lets choose the x-axis to be along the momentum of the kids.



Choose the origin to be the point where kids + man all stick together.

Note that the two kids have the same mass, $m_2 = m_1 = 30$ kg and the same speeds $v_2 = v_1 = 4.0$ m/s

In our coordinate system, their net y momentum is zero (since they travel along x-axis afterwards

Hence $P_{1y} = V_{2y}$

$m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2 \xrightarrow{m_1 = m_2,\ v_1 = v_2} \theta_1 = \theta_2 \equiv \theta$ (defines angle θ)

Net momentum (all in x) of the kids is

$$P_x = P_{1x} + P_{2x} = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

$$P_x = 2mv\cos\theta \quad \text{where} \quad m = 30 \text{ kg}, \ v = 4.0 \text{ m/s}$$

Since they and man stick together at rest, then the man's momentum (before collision) must be opposite this

$$\mathcal{P}_x = -MV \quad \text{where } M = 75 \text{ kg} \quad V = 2.0 \text{ m/s}$$

Together they have zero momentum

$$P_x + \mathcal{P}_x = 0 = 2mv\cos\theta - MV$$

$$\cos\theta = \frac{MV}{2mv} = \frac{(75)(2.0)}{2(30)(4.0)} = 0.625$$

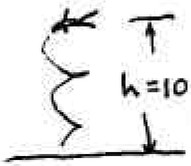
$$\theta = 51.3^\circ$$

So, angle between children is

$$2\theta = 2(51.3) = \underline{102.6^\circ}$$

11- 7 A diver makes 2.5 revolutions on the way from a 10-m-high platform to the water. Assuming zero initial vertical velocity, find the diver's average angular velocity during a dive.

11-7



Since the diver is in free-fall, and starts at rest

$$y = \frac{1}{2} g t^2 \qquad t = \sqrt{\frac{2y}{g}}$$

So, the time it takes to fall is $t_1 = \sqrt{\frac{2h}{g}}$

During $t = t_1$, the diver rotates through 2.5 revolutions or $\theta = 2.5(2\pi)$ radians

Hence $\omega = \frac{\theta}{t} = 2.5(2\pi)\sqrt{\frac{g}{2h}} = 2.5(2\pi)\sqrt{\frac{9.8}{2(10)}}$

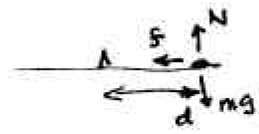
$\omega = 11 \frac{rad}{sec}$ average angular velocity

11-32 A record turntable is rotating at 33 1/3 rev/min. A watermelon seed is on the turntable 6.0 cm from the axis of rotation.

(a) Calculate the acceleration of the seed, assuming that it does not slip.
(b) What is the minimum value of the coefficient of static friction between the seed and the turntable if the seed is not to slip?
(c) Suppose that the turntable achieves its angular speed by starting from rest and undergoing a constant angular acceleration for 0.25 s. Calculate the minimum coefficient of static friction required for the seed not to slip during the acceleration period.

11-32

Looking edgewise at the seed with the forces.



The (static) friction force is
$f = \mu_s N = \mu_s mg$

(a) The acceleration of the seed is
$a_c = r\omega^2$
where $r = d = .06$ m
$\omega = 2\pi\left(33\tfrac{1}{3}\right)\frac{\text{rad}}{\text{min}} \times \frac{1\text{ min}}{60\text{ sec}} = 3.49\ \text{rad/s}$
So $a_c = (.06\text{ m})(3.49\ \text{rad/s})^2 = 0.73\ \text{m/s}^2$

(b) Since $f = m a_c$ or $\mu_s m g = m a_c$
$\mu_s = \frac{a_c}{g} = \frac{0.73}{9.8} = 0.075$ (for no slip)

(c) While getting up to speed, the total acceleration is
$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(0.73)^2 + \left[(.06)\left(\frac{3.49}{.25}\right)\right]^2} = 1.11\ \text{m/s}^2$

$a_t = r\alpha$
$a_t = d\,\frac{\omega}{t}$ with $t = .25$ s

$\mu_s = \frac{a}{g} = \frac{1.11}{9.8} = 0.11$ (for no slip)

11-44 Delivery trucks that operate by making use of energy stored in a rotating flywheel have been used in Europe. The trucks are charged by using an electric motor to get the flywheel up to its top speed of 200π rad/s. One such flywheel is a solid, uniform cylinder with a mass of 500 kg and a radius of 1.0 m.

(a) What is the kinetic energy of the flywheel after charging?
(b) If the truck operates with an average power requirement of 8.0 kW, for how many minutes can it operate between chargings?

11-44

(a) Kinetic Energy of Flywheel

Flywheel = cylinder rotating about axis ⊥ wheel through center

$I = \frac{1}{2} M R^2$

$K = \frac{1}{2} I \omega^2 = \frac{1}{2}\left(\frac{1}{2} M R^2\right)\omega^2$

$K = \frac{1}{4} M R^2 \omega^2 = \frac{1}{4}(500\,\text{kg})(1.0)^2(200\pi)^2$

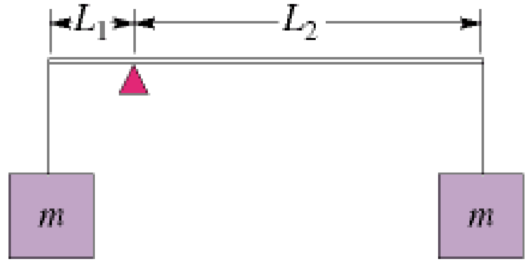
$K = 4.9 \times 10^7$ Joules

(b) Since $P = \frac{W}{t}$, i.e. power is work per unit time. Assume 100% efficiency in converting K to useful work

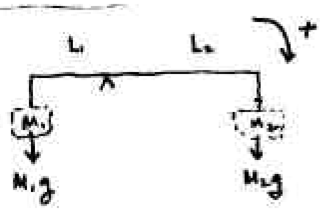
Then $t = \frac{K}{P} = \frac{4.9 \times 10^7 \text{ Joules}}{8.0 \times 10^3 \text{ Joules/s}} \times \frac{1 \text{ min}}{60 \text{ s}} = 103$ minutes

11-57 Figure 11-43 shows two blocks, each of mass m, suspended from the ends of a rigid massless rod of length $L_1 + L_2$, with L_1 = 20 cm and L_2 = 80 cm. The rod is held horizontally on the fulcrum and then released. What are the magnitudes of the initial accelerations of

(a) the block closer to the fulcrum and
(b) the other block?



11 – 57



Sketch and define (arbitrarily) positive rotation as clockwise.
We are asked about instantaneous angular acceleration as system is released (at instant pictured).

$\sum \tau = I\alpha$

Net torque is (about pivot) $M_2 g L_2 - M_1 g L_1$

System moment-of-inertia is $I = M_1 L_1^2 + M_2 L_2^2$

So $(M_2 L_2 - M_1 L_1) g = (M_1 L_1^2 + M_2 L_2^2)\,\alpha$

Since $M_1 = M_2 = m$ then masses cancel

$\alpha = g \dfrac{L_2 - L_1}{L_2^2 + L_1^2} = 9.8 \dfrac{0.8 - 0.2}{(0.8)^2 + (0.2)^2}$

$\alpha = 8.65$ rad/s

Since α is +ve, then system rotates clockwise and
a_2 = acceleration of M_2 is down
$a_2 = L_2 \alpha = (0.8)(8.65) = 6.92$ m/s²

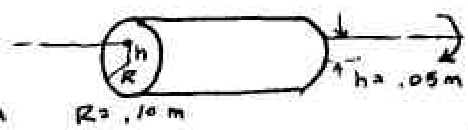
And a_1 = acceleration of M_1 is up
$a_1 = L_1 \alpha = (0.2)(8.65) = 1.73$ m/s²

11-64 A uniform cylinder of radius 10 cm and mass 20 kg is mounted so as to rotate freely about a horizontal axis that is parallel to and 5.0 cm from the central longitudinal axis of the cylinder.

(a) What is the rotational inertia of the cylinder about the axis of rotation?
(b) If the cylinder is released from rest with its central longitudinal axis at the same height as the axis about which the cylinder rotates, what is the angular speed of the cylinder as it passes through its lowest position?

11-64 Make picture

(a) Find I = moment of inertia about the rotation axis



Using parallel axis theorem $I = I_{com} + Mh^2$

and recognizing (for cylinder) $I_{com} = \frac{1}{2}MR^2$

$$I = \frac{1}{2}(20)(0.10)^2 + (20)(.05)^2 = 20\,[.00750]$$

$$I = 0.15 \text{ kg-m}^2$$

(b) If released with com at same height as axis, when it rotates to com h below axis, the potential energy has changed $U = Mgh$

So the kinetic energy gained is $K = \frac{1}{2}I\omega^2 = Mgh$

$$\omega = \sqrt{\frac{2Mgh}{I}} = \sqrt{\frac{2(20)(9.8)(.05)}{0.15}}$$

$$\omega = 11.4 \text{ rad/s}$$